

More or less?

Using CAS in teaching of Mathematics -
15 years of experience

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CADGME2016

1



PÉCSI TUDOMÁNYEGYETEM
♦ JUBILEUM 650 ♦

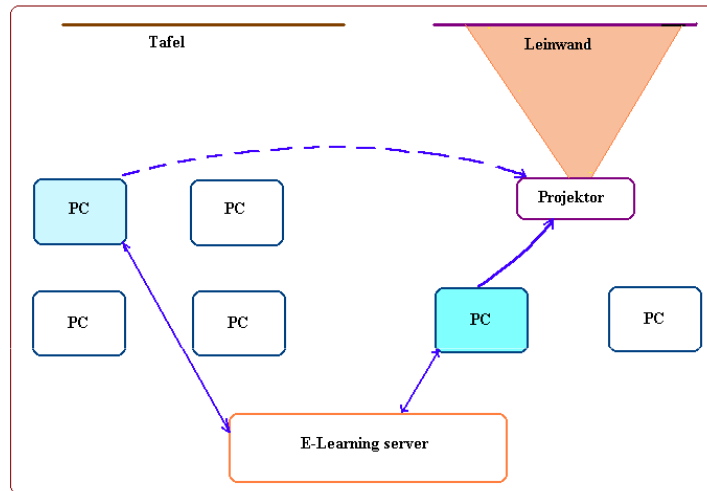


Hungarian King Louis the Great initiated establishment of a university
in the episcopal city of Pécs in 1367
Fresco of Andor Dudás in the Hall of University of Pécs (1923)

CADGME2016



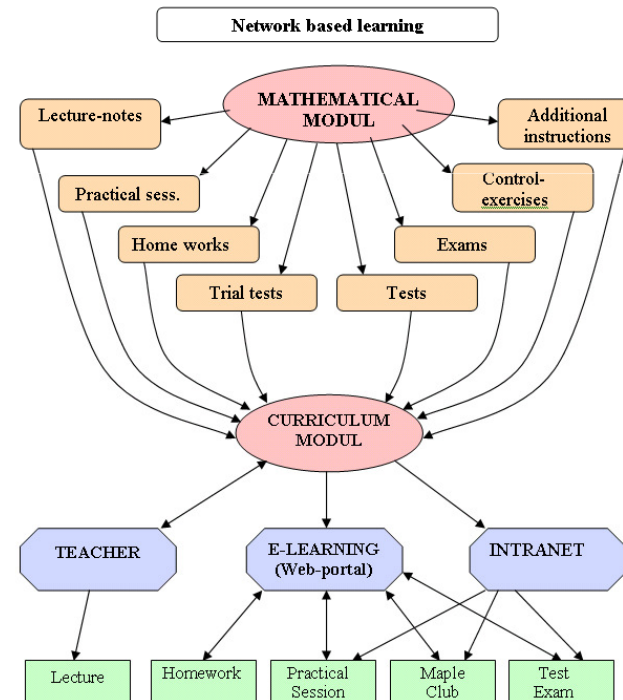
- **Experimental period** – first steps – new tool – revolution – new didactical access (CAS was new for teachers and students)
- **Discovering period** – pre-designed worksheets – usage as many times as possible (new for students)
- **Period of expanded use** – new didactical tasks – limits of utility – development of hardware and software – test and assessment systems – many CAS applications for mobile phone – integrating programming, engineering and Math courses (it is the part of every day life)

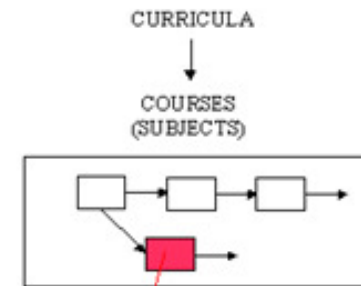
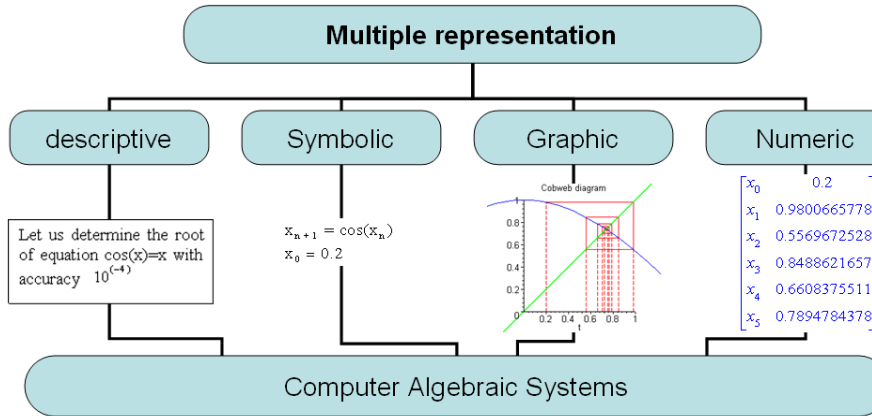


Time and human-consuming course

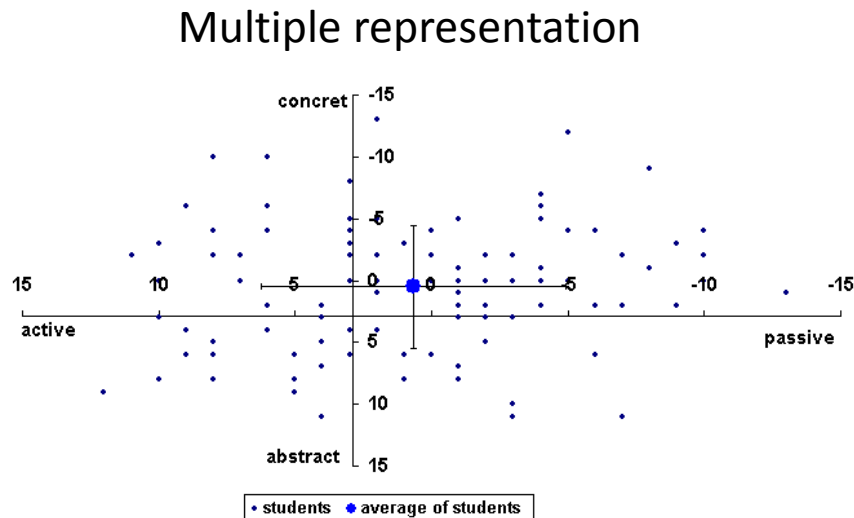
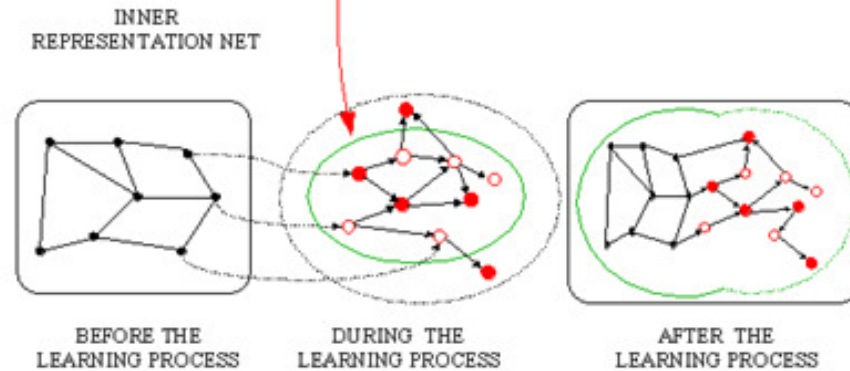
Same tasks with two groups- with and without CAS

Flexible data structures of CAS enable ambitious students to build-with their own mathematical microworlds





ENLARGING OF THE KNOWLEDGE REPRESENTATION NET

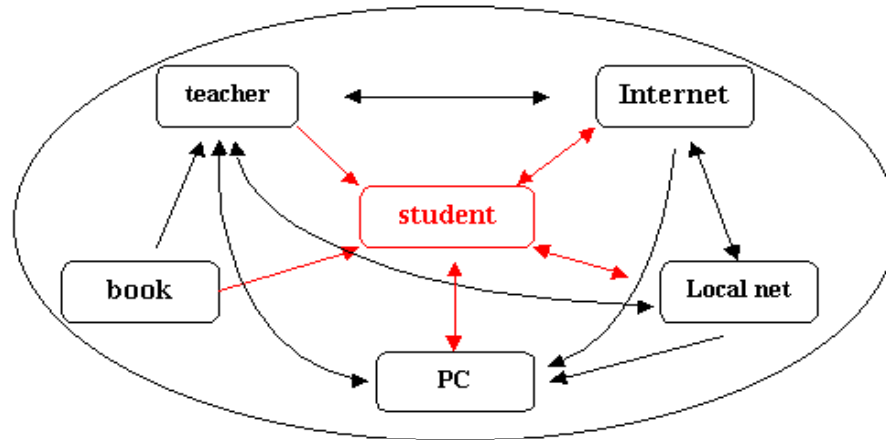


Learning style

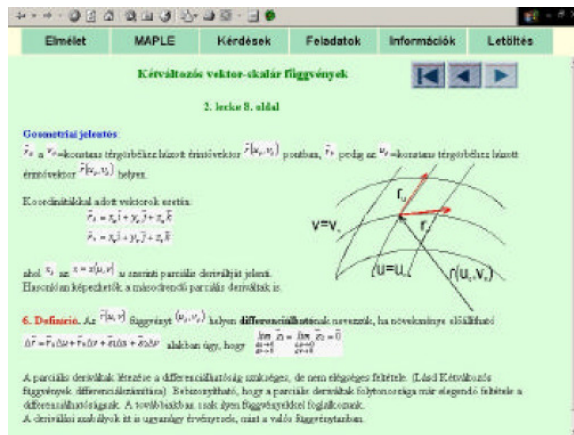
Knowledge representation NET (Enlarging)



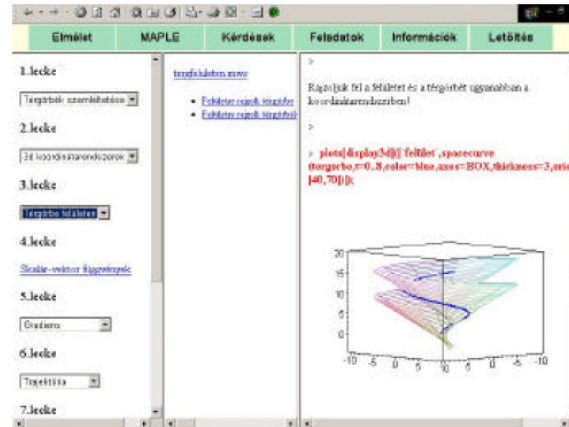
Experimental period



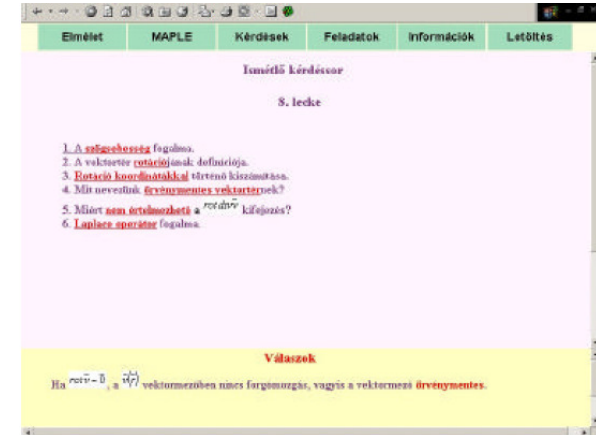
Interactive learning environment



Theory



Maple application



Test

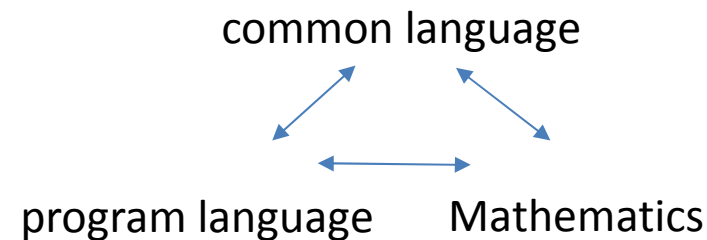


Expectations and observations:

- Students can become active participants in the learning-teaching process +/-
- Using the tools makes it possible to teach concepts which are often used in engineering +
- Extend creative learning +/-
- Structured knowledge-building – modularization +
- Multiple representation +
- Changing learning style of students passive \Rightarrow active, concrete \Rightarrow abstract +/-
- Development of conjecture –
- Easy visualization +

Difficulties:

- Inexperience
- Technical hadness 1D input
- Lack of time
- Didactical problems





PÉCSI TUDOMÁNYEGYETEM JUBILEUM 650

Discovering period

The screenshot shows the 'E-learning' website for Pécsi University of Applied Sciences. The page features a navigation menu on the left with links like 'Saját menü', 'Hirdetéslista', 'SysTAMah', and 'Fájlok listája'. The main content area is divided into several columns, each containing a list of course materials or announcements, such as 'Egyelőleges', 'Hospitál szabály', and 'Dykonlat'. The interface is designed for easy navigation and access to educational resources.

The screenshot displays the NEPTUN (National Educational Portal) interface. At the top, there's a header with the NEPTUN logo and navigation tabs like 'Tanulmányi rendszer', 'Neptun Meet Street', 'Saját adatok', 'Virtuális tér', 'Dokumentumtár', 'Beállítások', 'Feladatok', 'Naptár', 'Hírek', and 'Kommunikáció'. The main area shows a list of documents for 'Mathematics A/2'. Below this, there's a detailed table of documents with columns for 'Dokumentum neve', 'Új Leírás', 'Méret', 'Feltöltő', 'Utolsó módosítás dátuma', 'Hozzárendelés dátuma', 'Letöltések', 'Publikus', and 'Kijelölés'. The table lists various documents like 'Syllabus', 'Lectures', and 'Practices' with their respective sizes and upload dates.

| Dokumentum neve | Új Leírás | Méret | Feltöltő | Utolsó módosítás dátuma | Hozzárendelés dátuma | Letöltések | Publikus | Kijelölés |
|------------------------------|---------------|----------|--|-------------------------|----------------------|------------|-------------------------------------|--------------------------|
| Syllabus | Mathematic... | 634 kB | Pérgésiné Dr. Hámosi Ildikó Viktória Dr. | 2016.02.07. 19:02:45 | 2016.01.30. 15:37:34 | 32 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Lectures | | 21149 kb | Pérgésiné Dr. Hámosi Ildikó Viktória Dr. | 2016.04.24. 17:44:52 | 2016.01.31. 20:22:34 | 88 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| application of integral | | 109 | mw | 2016.03.22. 12:51:34 | | 6 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| approximate integral | | 1232 | mw | 2016.03.22. 12:51:56 | | 5 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Applications of derivatives2 | | 1542 | pdf | 2016.02.07. 19:01:31 | | 18 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| properties of the functions | | 205 | mw | 2016.02.07. 19:01:42 | | 12 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Definite_ integral | | 4204 | ppbx | 2016.03.22. 12:52:12 | | 5 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Definite_ integral | | 3546 | pdf | 2016.04.06. 9:36:05 | | 4 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Functions of two variables | | 2761 | pdf | 2016.04.06. 9:36:14 | | 5 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| function of two variables1 | | 143 | mw | 2016.04.03. 14:51:55 | | 3 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Functions of two variables | | 3817 | ppbx | 2016.04.03. 14:51:39 | | 4 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Indefinite_integral | | 1834 | pdf | 2016.02.14. 18:20:09 | | 16 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Differential equations | | 1756 | pdf | 2016.04.24. 17:44:44 | | 5 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Practices | Applicatio... | 7002 kb | Pérgésiné Dr. Hámosi Ildikó Viktória Dr. | 2016.04.24. 17:51:59 | 2016.01.31. 20:23:07 | 96 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Literature | | 19528 kb | Pérgésiné Dr. Hámosi Ildikó Viktória Dr. | 2016.02.07. 19:04:05 | 2016.02.07. 19:03:49 | 36 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Test results | | 122 kb | Pérgésiné Dr. Hámosi Ildikó Viktória Dr. | 2016.05.10. 13:31:01 | 2016.03.09. 16:20:03 | 45 | <input checked="" type="checkbox"/> | <input type="checkbox"/> |

The screenshot shows two interfaces side-by-side. On the left is the 'e-tr' interface with a user profile for 'Perjesiné Hámosi Ildikó' and a list of 'My scenes' including various course identifiers like 'DPHMK_DKINF' and 'DPHMKAN0013'. On the right is the 'CooSpace' interface with the text '(CooSpace) Az oktatási kooperáció virtuális tere!' and a search bar.

Network based learning
Cover the whole syllabus of the course



From the Newton-law:

> `eq:=diff(x(t),t$2)+2*beta*diff(x(t),t)+omega^2*x(t)=0;`

$$eq := \left(\frac{d^2}{dt^2} x(t) \right) + 2 \beta \left(\frac{d}{dt} x(t) \right) + \omega^2 x(t) = 0$$

It's seen that this equation for $x(t)$ is a second-order, constant-coefficient, linear, homogenous differential equation system.

> `gen_sol:=dsolve(eq,x(t));`

$$gen_sol := x(t) = _C1 e^{((- \beta + \sqrt{\beta^2 - \omega^2}) t)} + _C2 e^{((- \beta - \sqrt{\beta^2 - \omega^2}) t)}$$

Let's check the shape of the given path-time function! Find the particular solution when $x(0)=0$, $D(x)(0)=vmax$.

> `part_sol:=dsolve({eq,x(0)=0,D(x)(0)=vmax},x(t));`

$$part_sol := x(t) = \frac{1}{2} \frac{vmax e^{((- \beta + \sqrt{\beta^2 - \omega^2}) t)}}{\sqrt{\beta^2 - \omega^2}} - \frac{1}{2} \frac{vmax e^{((- \beta - \sqrt{\beta^2 - \omega^2}) t)}}{\sqrt{\beta^2 - \omega^2}}$$

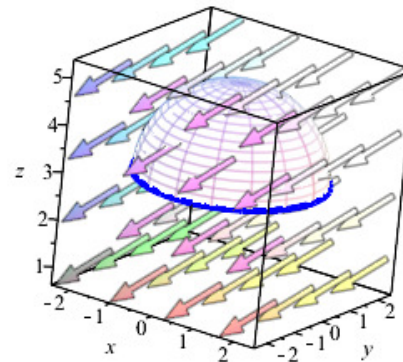
Instrumental orchestration

- Have short textual explanations
- Introduce first every new command - according to the principle of spirality - through a mathematical problem



Surface, curve, field together

```
> abra4:=proc(mezo, felulet, gorbe, u1, u2, v1, v2, t1, t2)
    local rot_mezo, rotabra, feluletabra, gorbeabra;
    rot_mezo:=curl(mezo, [x, y, z]);
    rotabra:=fieldplot3d(rot_mezo, x=-2..2, y=-2..2, z=1..5, grid=[4,4,4], arrows=THICK, orientation=
[-48,40]):rotabra;
    feluletabra:=plot3d(felulet, u=u1..u2, v=v1..v2, axes=boxed,
orientation=[6,49], style=hidden, scaling=constrained, numpoints=1000):feluletabra;
    gorbeabra:=spacecurve(gorbe, t=t1..t2, color=blue, axes=BOX, thickness=4, orientation=[-30,50])
:gorbeabra;
    plots[display3d]([feluletabra, rotabra, gorbeabra], orientation=[-48,40]);
end:
> abra4(vektormezo, felgomb, kor, 0, 2*Pi, 0, Pi/2, 0, 2*Pi);
```



Applications being written according with gradation (white box – black box)



```
> restart;with(VectorCalculus):
> SetCoordinates('cartesian'[x,y,z]);
> field:=VectorField(<x,-z,2*z>);
> curve:=<r*cos(t),r*sin(t),t*h/(2*Pi)>;
```

*cartesian*_{x,y,z}

$$field := (x)e_x - ze_y + 2ze_z$$

$$curve := (r \cos(t))e_x + (r \sin(t))e_y + \frac{1}{2} \frac{t h}{\pi} e_z$$

```
> LineInt(field,Path(curve,t=0..2*Pi),'inert')=LineInt(field,Path(curve,t=0..2*Pi));
```

$$\int_0^{2\pi} \left(-r^2 \cos(t) \sin(t) - \frac{1}{2} \frac{t h r \cos(t)}{\pi} + \frac{1}{2} \frac{t h^2}{\pi^2} \right) dt = h^2$$

```
> Lineintegral:=proc(mezo,gorbe,t1,t2)
  local erinto,skalarszorzat,integrandus,integral,lok;
  erinto:=diff(gorbe,t);
  lok:=subs({x=gorbe[1],y=gorbe[2],z=gorbe[3]},mezo);
  skalarszorzat:=DotProduct(erinto,lok);
  integrandus:=simplify(skalarszorzat);
  integral:=Int(integrandus,t=t1..t2)=int(integrandus,t=t1..t2);
end:
> Lineintegral(field, curve,0,2*Pi);
```

$$\int_0^{2\pi} \left(-\frac{1}{2} \frac{2r^2 \cos(t) \sin(t) \pi^2 + t h r \cos(t) \pi - t h^2}{\pi^2} \right) dt = h^2$$

Instrumental orchestration

- Step by step \Rightarrow self made procedures \Rightarrow built in procedures

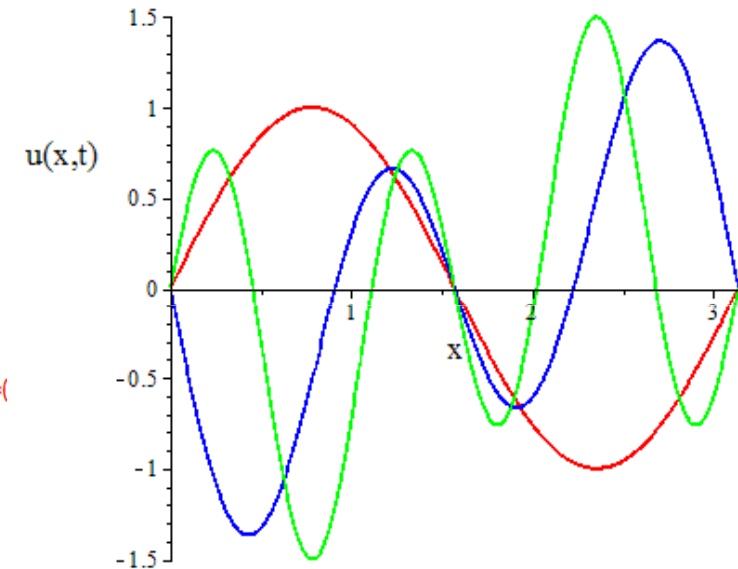


Discovering period

Black box: if we have no time, or students have only few knowledge of the solution of the differential equation, and Fourier series

White box: after the detailed explanation, for experiments, or solving more complicated problem

```
> restart; with(VectorCalculus): with(plots):  
> Cord_Eqn := { diff( u(x,t), t, t) = Laplacian( u(x,t), 'cartesian'[x]) };  
Cord_Eqn := {  $\frac{\partial^2}{\partial t^2} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$  }  
> sol:-animate( [[u(x,t) , color=red],  
                [-u(x,t)/2-sin(4*x) , color=blue], [-u(x,t)/2+sin(6*x) , color=green]], t=(  
                labels=["x", "u(x,t)"], labelfont=[TIMES,ROMAN,14],  
                scaling=constrained);
```



Instrumental orchestration

Black box white box: visualization, engineering applications



Expectations and observations

- Network based learning +
- Cover the whole syllabus of the course +/-
- Usage as many times as possible +/-
- Instrumental orchestration
 - Have short textual explanations +
 - Introduce first every new command - according to the principle of spirality - through a mathematical problem +
 - Applications being written according with gradation (white box – black box) +
 - Step by step \Rightarrow self made procedures \Rightarrow built in procedures +
 - Black box white box: visualization, engineering applications +

Difficulties:

- Deep understanding only for the best students
- Everything is ready: no conceptual understanding
- Didactical problems:
 - Some exercises became routine ones with help of it
 - Not the technical details but the mathematical meaning is always the most important
 - Avoid using CAS only for the end in itself; it is only the inferior of the mathematical subject matter



| | | |
|----------------------------|--|---------------------------------|
| $\int f dx$ | $\int_a^b f dx$ | $\sum_{i=k}^n f$ |
| $\prod_{i=k}^n f$ | $\frac{d}{dx} f$ | $\frac{\partial}{\partial x} f$ |
| $\lim_{x \rightarrow a} f$ | $a+b$ | $a-b$ |
| $a \cdot b$ | $\frac{a}{b}$ | a^b |
| a_n | a_* | \sqrt{a} |
| $\sqrt[n]{a}$ | $a!$ | $ a $ |
| e^a | $\ln(a)$ | |
| $\log_{10}(a)$ | $\log_b(a)$ | |
| $\sin(a)$ | $\cos(a)$ | $\tan(a)$ |
| $\binom{a}{b}$ | $f(a)$ | $f(a,b)$ |
| | $f := a \rightarrow y$ | |
| | $f := (a,b) \rightarrow z$ | |
| $f(x) _{x=a}$ | $\begin{cases} -x & x < a \\ x & x \geq a \end{cases}$ | |

Printed: $\int_2^3 \frac{\sin x}{\cos^2 x} + \sqrt[3]{x} dx$

> $\int_2^3 \frac{\sin(x)}{\cos(x)^2} + \sqrt[3]{x} dx$

> `int(sin(x)/cos(x)^2+x^(1/3), x = 2 .. 3);`

>

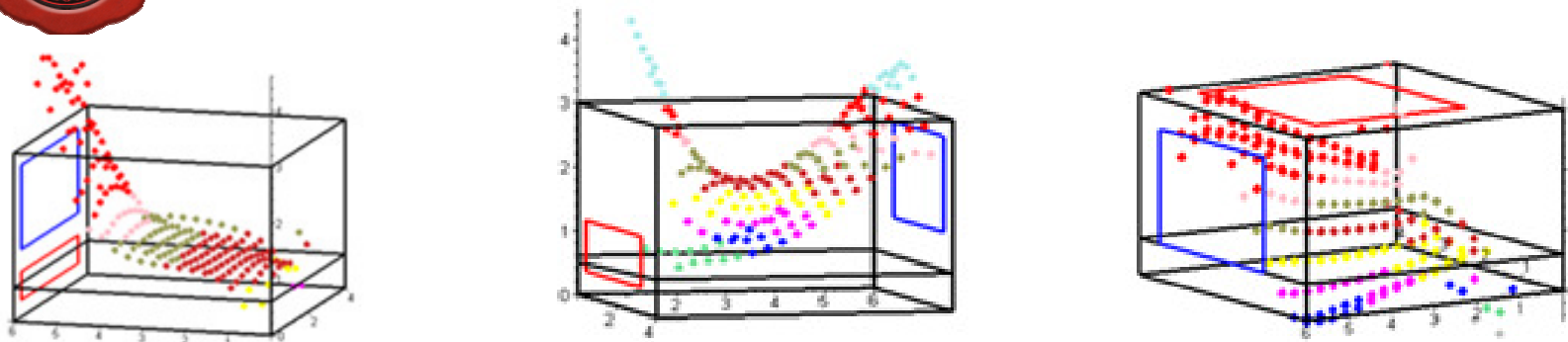
User friendly interface: it is not necessary to prepare everything

Lecture: presentation (ppt, Prezi, video...) definition, theorems, few example + oral explanation

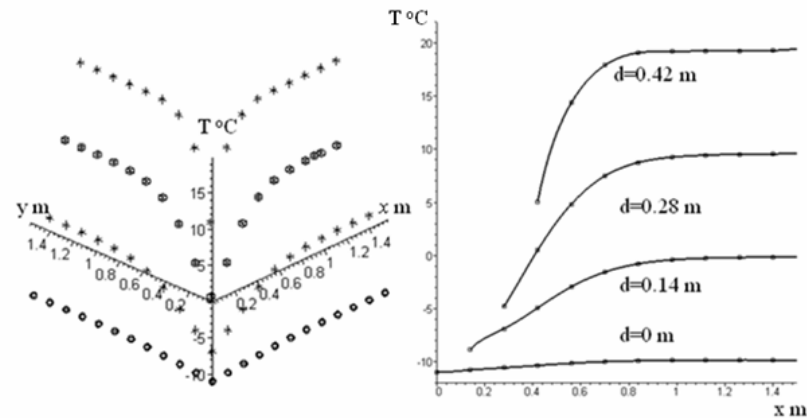
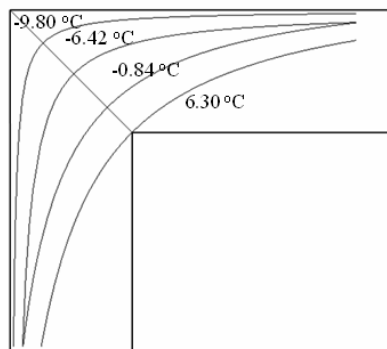
Seminar: paper work, simple examples , more complicated examples using CAS, independent student work

[Prepared worksheet for practice class](#)

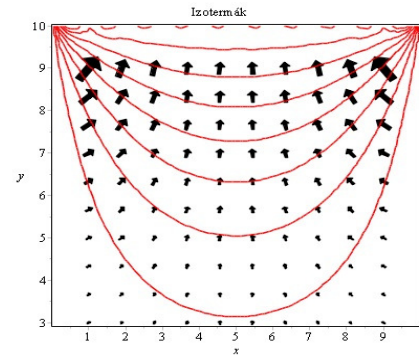
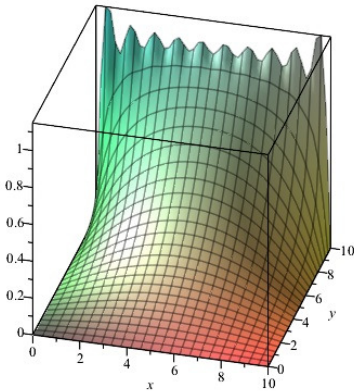
[One expected solution](#)



I. Perjési- Hámori: Simulation of Heat Radiation Asymmetry With Maple 7th Vienna Conference on Mathematics Modelling Febr. 15-17, 2012



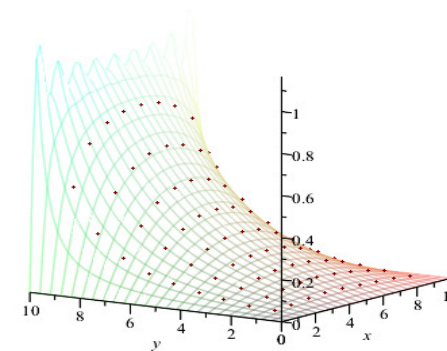
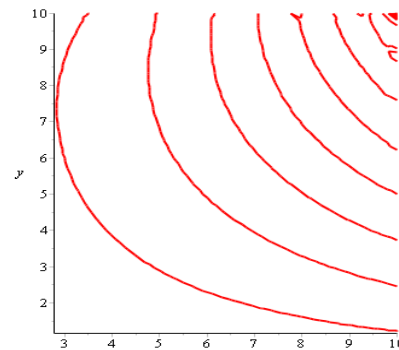
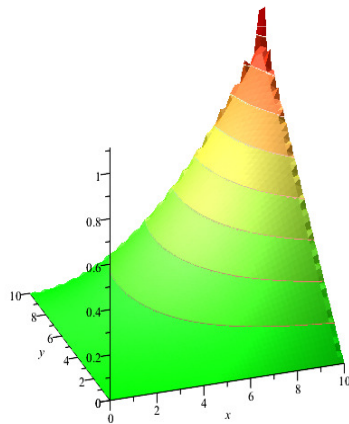
J. Vajda, I. Perjési-Hámori: *Two dimensional mathematical model of heat-transmission of one- and double-layer building* Pollack Periodica Vol. 2, No.3, pp.25-34, 2007.



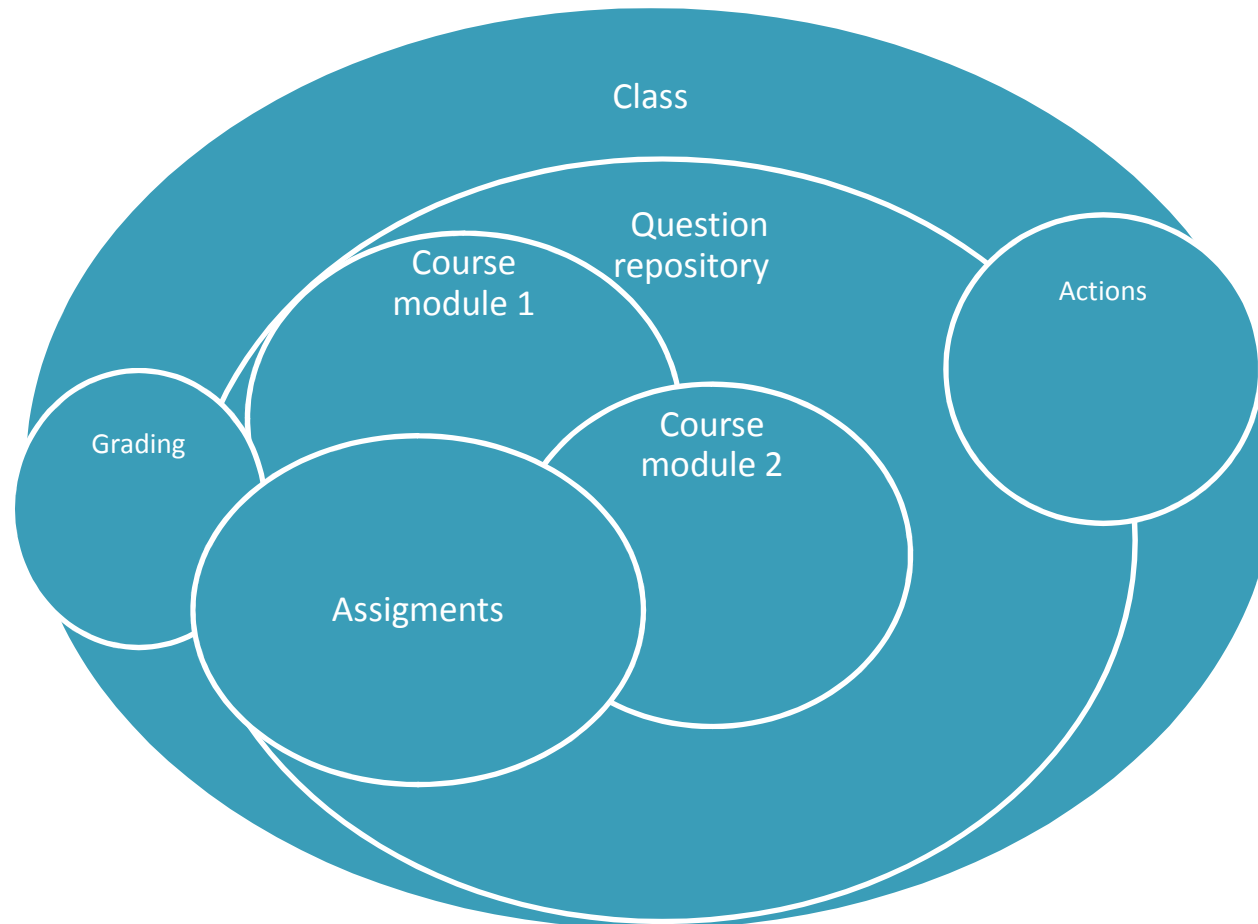
```

> elliptic := proc (n, m, a, b, c, d, f, g)
local i, j;
global l, e, u, h, k;
for j to m-1 do
for i to n-1 do
l := (j-1)*(m-1) + i;
h := (b-a)/n; k := (d-c)/m;
if l mod (n-1) = 1 then u[l-1] := g(a, c+j*k) fi;
if l mod (n-1) = 0 then u[l+1] := g(b, c+j*k) fi;
if l-n+1 <= 0 then u[l-n+1] := g(a+i*h, c) fi;
if l+n-1 > (n-1)*(m-1) then u[l+n-1] := g(a+i*h, d) fi;
e[l] := k^2*(u[l-1]-2*u[l]+u[l+1]) + h^2*(u[l-(n-1)]-2*u[l]+u[l
+(n-1)]) = f(a+i*h, c+j*k);
unassign('u[l-1]','u[l]','u[l+1]','u[l-n+1]','u[l+n-1]');
od;
od;
end proc;

```



I. Perjési-Hámori: Two Dimensional Mathematical Model of Heat-transmission Using MAPLE poster 8th Vienna Conference on Mathematics Modelling Febr. 17-20, 2015 Mathematical Modelling , Volume # 8 | Part# 1 689-690





Matematika3

PTE

Ildikó Perjesiné Hámori (perjesi@pmmik.pte.hu)

Select the link for an assignment to begin:

| Assignment Name | Points | Type | Availability |
|---|--------|---------------|-----------------------|
| Kétféltözös integrál - feladat | 24.0 | Homework/Quiz | Unlimited |
| Kétféltözös integrál elmélet | 8.0 | Homework/Quiz | Unlimited |
| Kétféltözös függvény gradiens és szélsőérték- feladatok | 31.0 | Homework/Quiz | Unlimited |
| Kétféltözös függvény parciális és iránymenti derivált-feladatok | 32.0 | Homework/Quiz | Unlimited |
| Kétféltözös függvények deriválása-elmélet | 5.0 | Homework/Quiz | Unlimited |
| Függvény sor elmélet | 10.0 | Homework/Quiz | Unlimited |
| Függvény sorok gyakorló feladatok | 12.0 | Homework/Quiz | Unlimited |
| Szamsorok elmelet gyakorlo | 10.0 | Homework/Quiz | Unlimited |
| Szamsoros feladatok gyakorlo | 30.0 | Homework/Quiz | After 4/26/13 9:42 AM |

Question 4: (1 points)

A T tartományon integrálható $f(x, y)$ kétféltözös függvény $t(T) \neq 0$ területű T tartományra vonatkozó integrálközepértékén az $\frac{1}{t(T)} \cdot \iint_T f(x, y) dx dy$ kifejezéssel definiált számot értjük.

Határozza meg az $f(x, y) = x + 2y + 2$ függvény T tartományra vonatkozó integrálközepértékét, ha a tartományt az x -tengely, az $x = 4$ egyenes és a $g(x) = 2\sqrt{x}$ függvény grafikonja határolja.

$t(T) =$

A belső integrál értéke:

A kettős integrál értéke:

Az integrálközepérték:



Question 2: (1 points)

Válassza ki az alábbi tartományok közül azokat, amelyek esetén az $\iint_T f(x, y) \, dT$ integrál az $f(x, y) = x^2 + 4y$ függvény grafikonja és a T tartomány által határolt hengeres térrész térfogatának

számértékét adja.

- $-2 \leq x \leq 2$ és $-1 \leq y \leq 1$
- $-2 \leq x \leq 2$ és $0 \leq y \leq 1$
- $y = \sqrt{x}$ és $y = 2x - 1$
- $y = 4x^2$ és $y = x + 5$ görbék által határolt tartomány.

How did I do?

Comment:

Your response

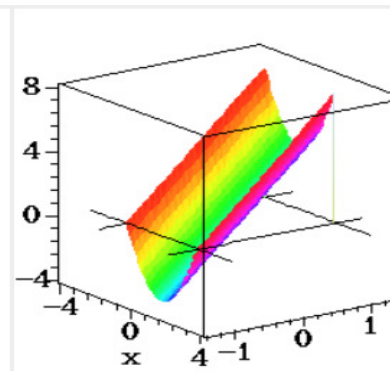
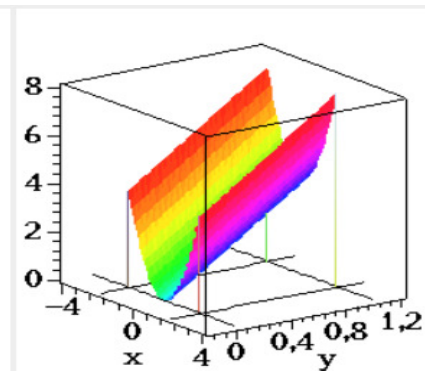
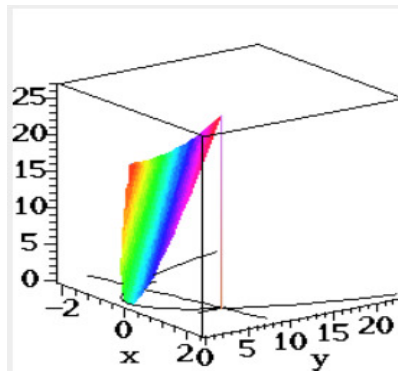
Válassza ki az alábbi tartományok közül azokat, amelyek esetén az $\iint_T f(x, y) \, dT$ integrál az $f(x, y) = x^2 + 4y$ függvény grafikonja és a T tartomány által határolt hengeres térrész térfogatának számértékét adja.

0% (0%)

Correct response

Válassza ki az alábbi tartományok közül azokat, amelyek esetén az $\iint_T f(x, y) \, dT$ integrál az $f(x, y) = x^2 + 4y$ függvény grafikonja és a T tartomány által határolt hengeres térrész térfogatának számértékét adja.

$-2 \leq x \leq 2$ és $0 \leq y \leq 1$, $y = 4x^2$ és $y = x + 5$ görbék által határolt tartomány.





MathPapa

Problem:

Solve $x + y = 7$; $x + 2y = 11$

Steps:

I will try to solve your system of equations.

$$x + y = 7; x + 2y = 11$$

Step: Solve $x + y = 7$ for x :

$$x + y + -y = 7 + -y \text{ (Add } -y \text{ to both sides)}$$

$$x = -y + 7$$

Step: Substitute $-y + 7$ for x in $x + 2y = 11$:

$$x + 2y = 11$$

$$-y + 7 + 2y = 11$$

$$y + 7 = 11 \text{ (Simplify both sides of the equation)}$$

$$y + 7 + -7 = 11 + -7 \text{ (Add } -7 \text{ to both sides)}$$

$$y = 4$$

Step: Substitute 4 for y in $x = -y + 7$:

$$x = -y + 7$$

$$x = -4 + 7$$

$$x = 3 \text{ (Simplify both sides of the equation)}$$

Symbolab

$$\int \arctan(x) dx$$

Go

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Solution

$$\int \arctan(x) dx = x \arctan(x) - \frac{1}{2} \ln|x^2 + 1| + C$$

« Hide Steps

Steps

$$\int \arctan(x) dx$$

Apply Integration By Parts: $\int u'v = uv - \int u'v$
 $u = \arctan(x), u' = \frac{1}{x^2 + 1}, v' = 1, v = x$

Show Steps

$$= \arctan(x)x - \int \frac{1}{x^2 + 1} x dx$$

$$= x \arctan(x) - \int \frac{x}{x^2 + 1} dx$$

$$\int \frac{x}{x^2 + 1} dx = \frac{1}{2} \ln|x^2 + 1|$$

Show Steps

$$= x \arctan(x) - \frac{1}{2} \ln|x^2 + 1|$$

Add a constant to the solution ⓘ

$$= x \arctan(x) - \frac{1}{2} \ln|x^2 + 1| + C$$



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Period of expanded use



integrate sin(x)cos(x)^2



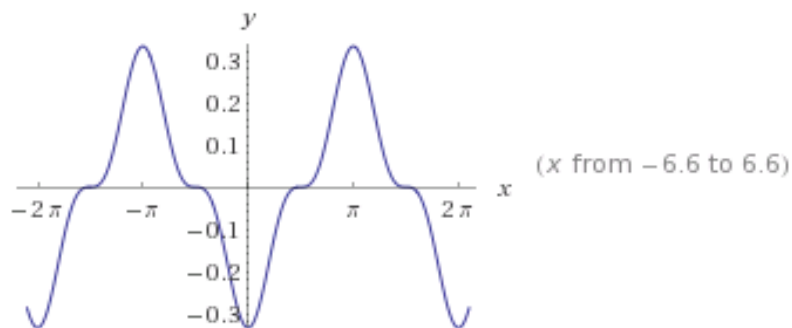
Web Apps Examples Random

Indefinite integral:

Step-by-step solution

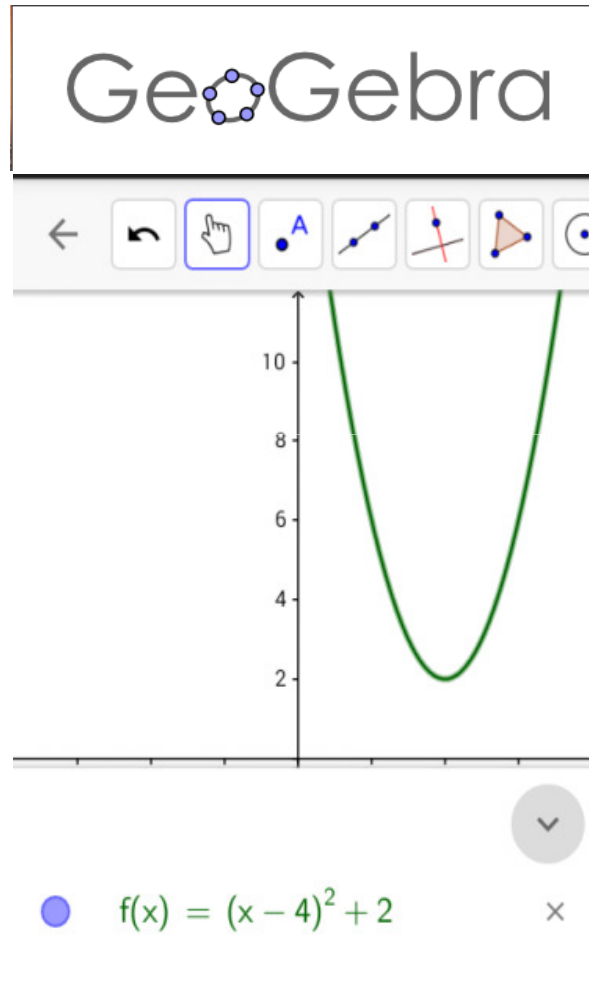
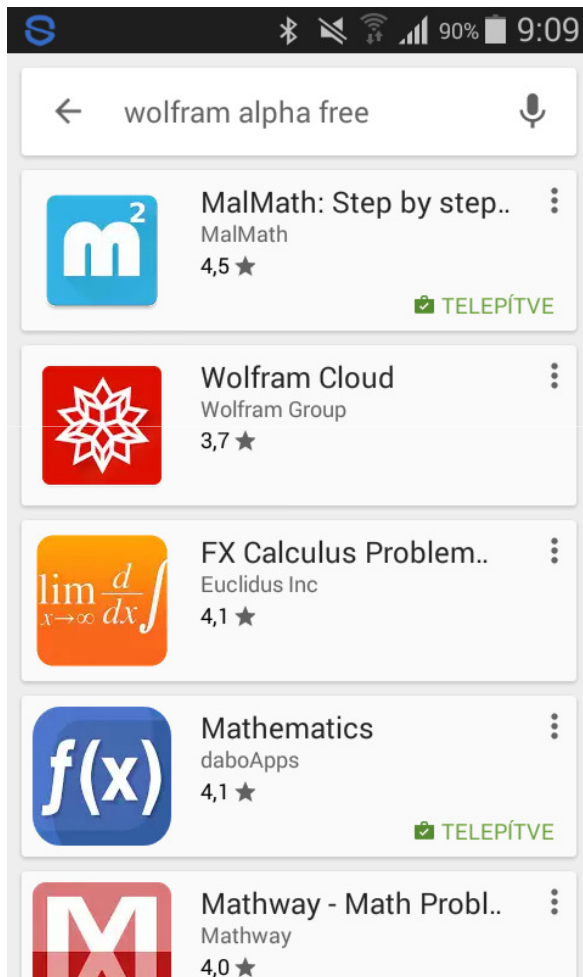
$$\int \sin(x) \cos^2(x) dx = -\frac{1}{3} \cos^3(x) + \text{constant}$$

Plots of the integral:



Enable interactivity

Enlarge | Data | Customize | Plaintext | Interactive



MathsTools app interface showing the Functions Integrator section. The function $f(x) = 1/(x^2+1)$ is entered, and the interval $a = -3$ and $b = 3$ is specified. Buttons for 'Plot f(x)' and 'Clear Graphic' are visible.



Expectations and observations:

- Students are aboriginals, teachers are immigrants in IT +/-
- Students are users but do not know about programming -
- User friendly interface +
- Test and assessment system based on Maple +/-
- CAS applications for mobile phone, free software's (GeoGebra) +/-
- Integrating programming, engineering and math courses (it is the part of every day life) +

Difficulties:

- Didactical problems:
 - Role of teacher is not clear
 - Why we have to understand math, why is not enough the applications?
 - There are standards in the softwares, which one is the more useful? (Price, university licenses, comparison)



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Conclusion

Usage depends on

- Students
- Topics
- Tutor

No comfortable, universal solution.

We have to find the appropriate application forms.

**The more we learn more we know about the
weakness of our knowledge.**



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Thank you for your attention