PÉCSI TUDOMÁNYEGYETEM
JUBILEUM 650
UNIVERSITY OF PECS JUBILEE

More or less?
Using CAS in teaching of Mathematics - 15 years of experience

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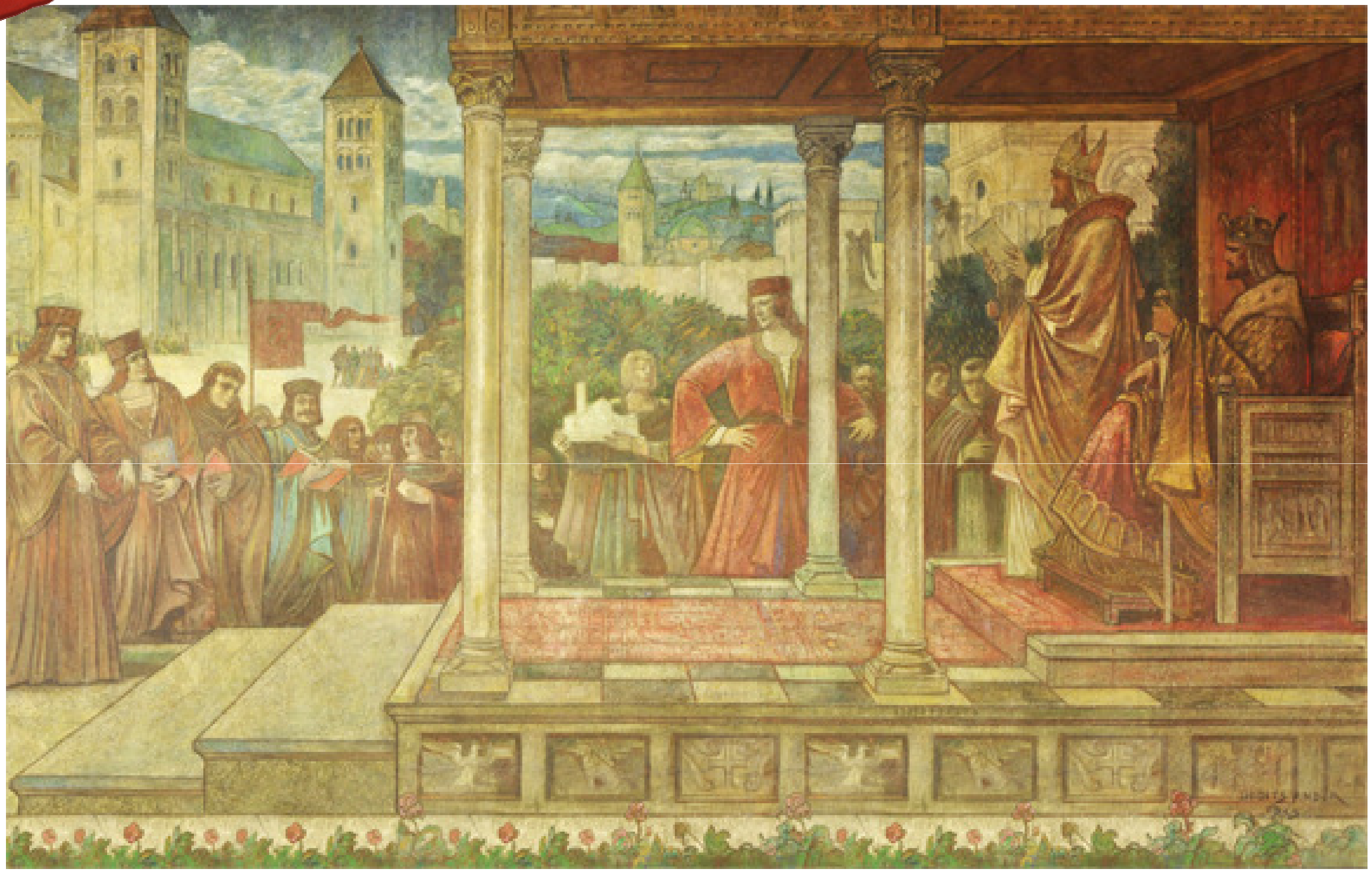
Pécsi Tudományegyetem
JUBILEUM 650



Hungarian King Louis the Great initiated establishment of a university
in the episcopal city of Pécs in 1367
Fresco of Andor Dudás in the Hall of University of Pécs (1923)

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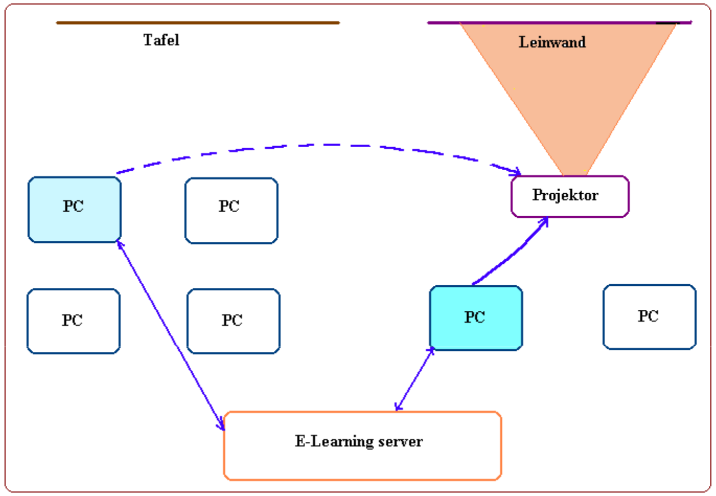
Outline

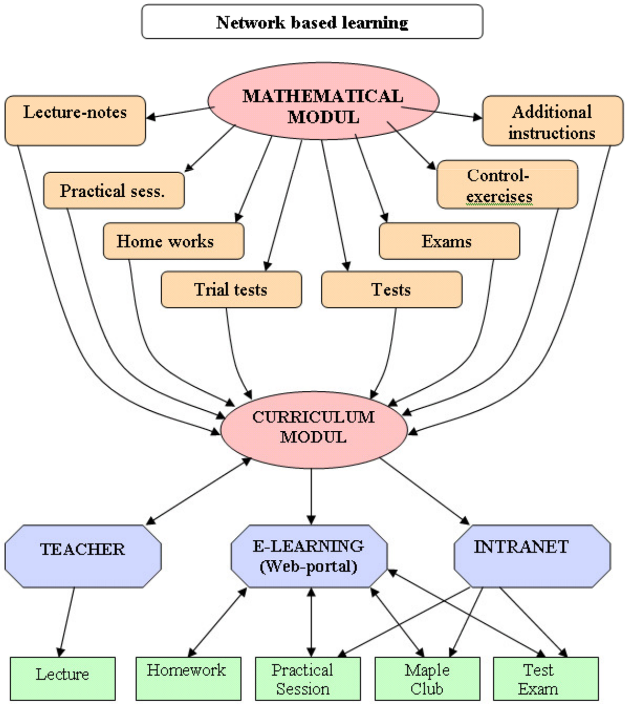
- **Experimental period** – first steps – new tool – revolution –new didactical access (CAS was new for teachers and students)
- **Discovering period** – pre-designed worksheets – usage as many times as possible (new for students)
- **Period of expanded use**– new didactical tasks – limits of utility – development of hardware and software – test and assessment systems – many CAS applications for mobile phone –integrating programing, engineering and Math courses (it is the part of every day life)

Experimental period



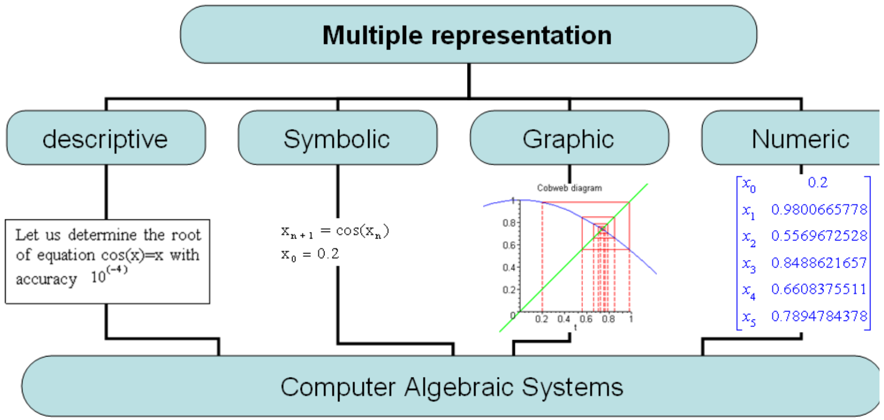
Time and human-consuming course

Same tasks   with two groups- with and without CAS

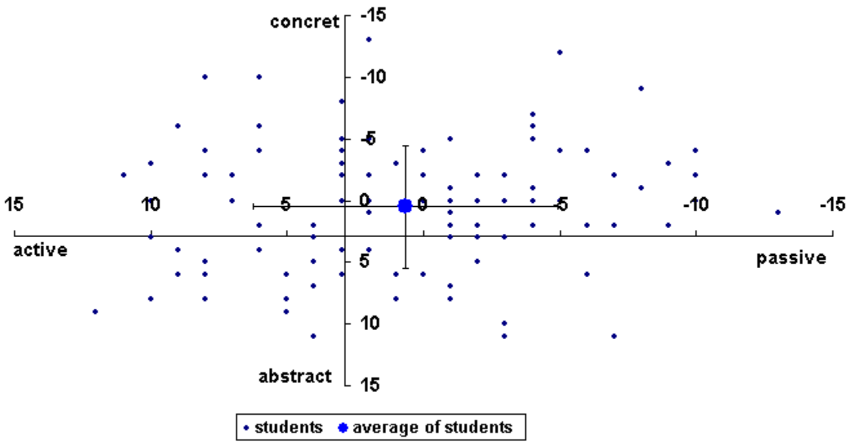


Flexible data structures of CAS enable ambitious students to build-with their own mathematical microworlds

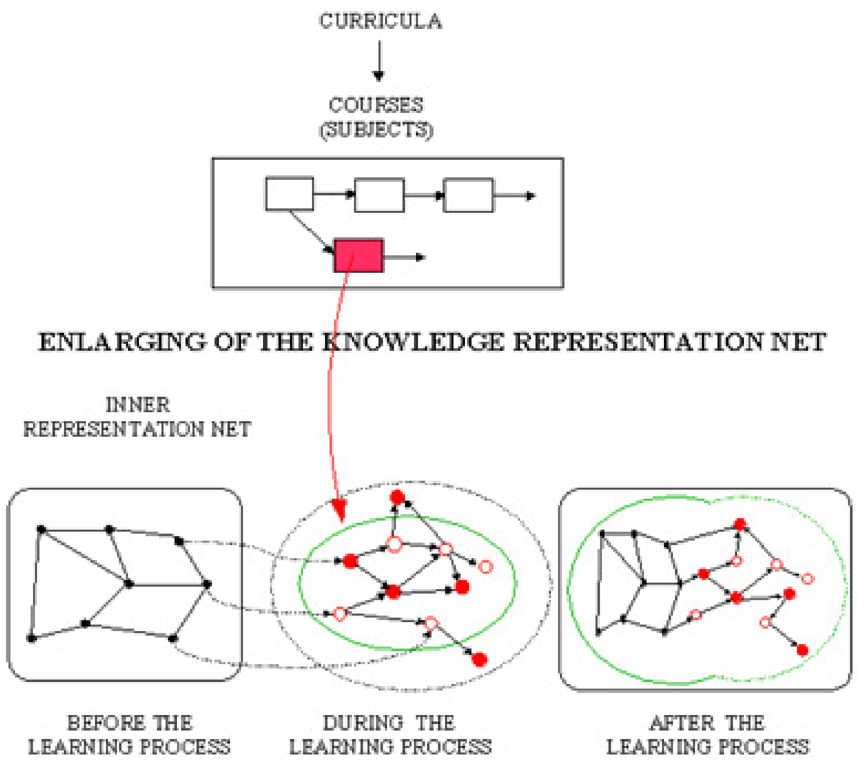
Experimental period



Multiple representation

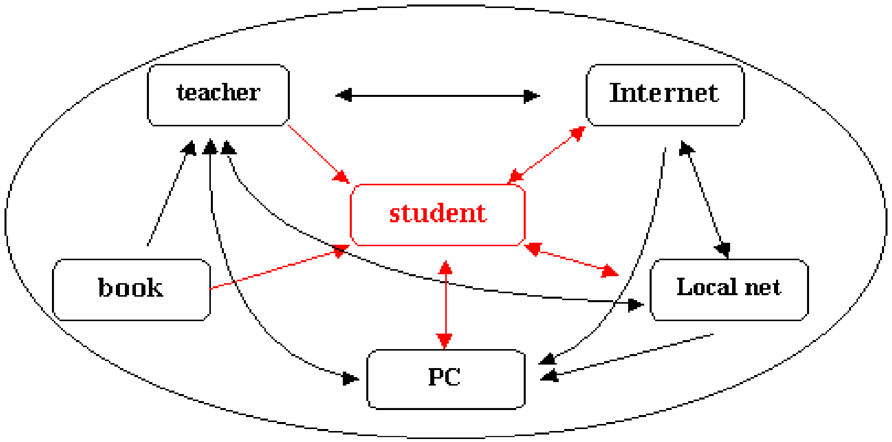


Learning style

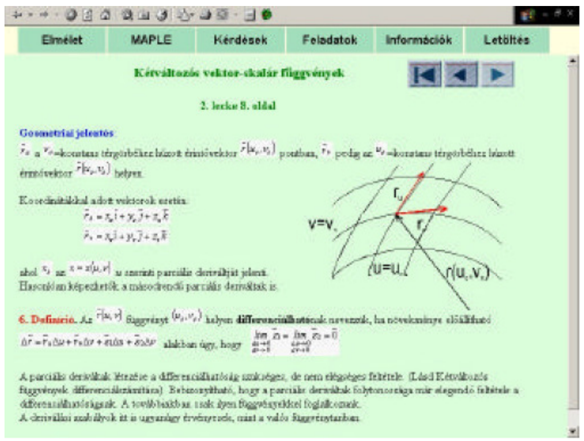


Knowledge representation NET (Enlarging)

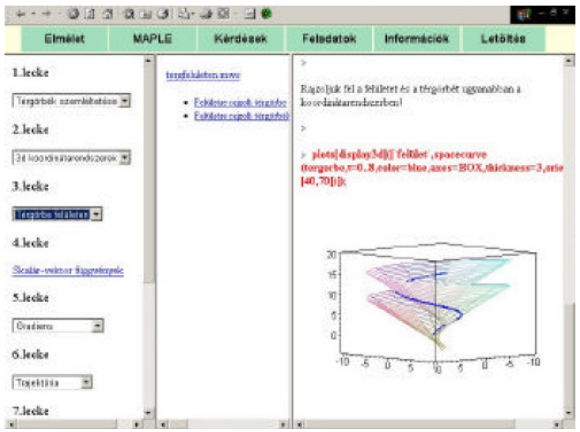
Experimental period



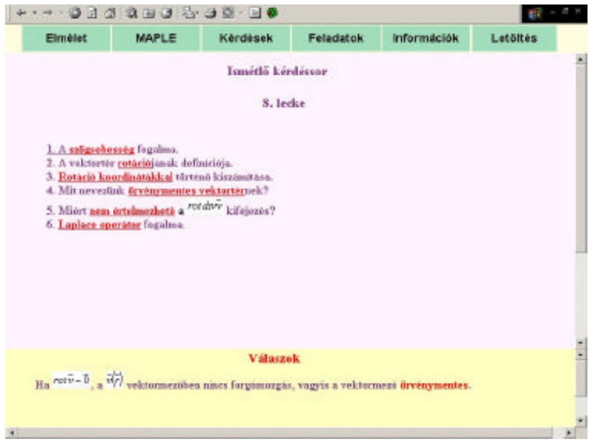
Interactive learning environment



Theory



Maple application



Test

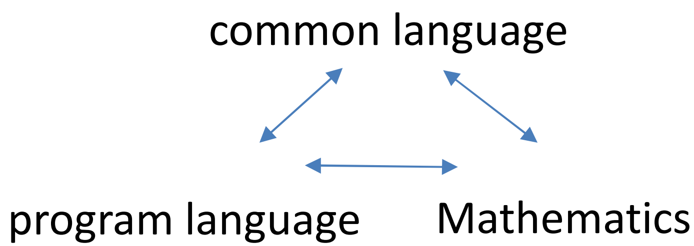
Experimental period

Expectations and observations:

- Students can become active participants in the learning-teaching process +/-
- Using the tools makes it possible to teach concepts which are often used in engineering +
- Extend creative learning +/-
- Structured knowledge-building – modularization +
- Multiple representation +
- Changing learning style of students passive ⇒active, concrete ⇒abstract +/-
- Development of conjecture –
- Easy visualization +

Difficulties:

- Inexperience
- Technical hadness 1D input
- Lack of time
- Didactical problems



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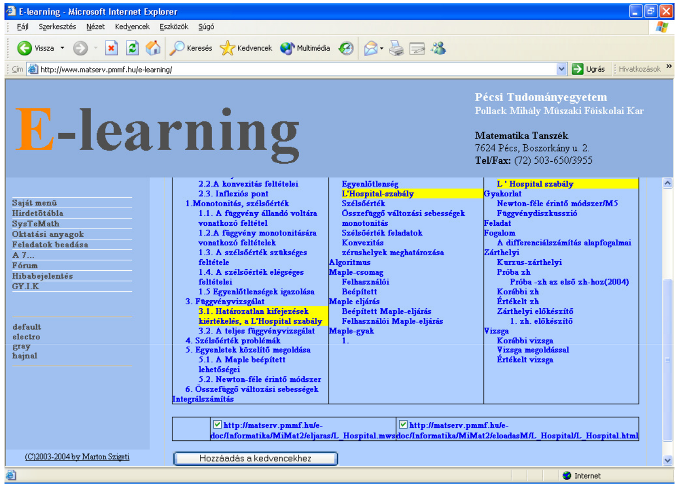
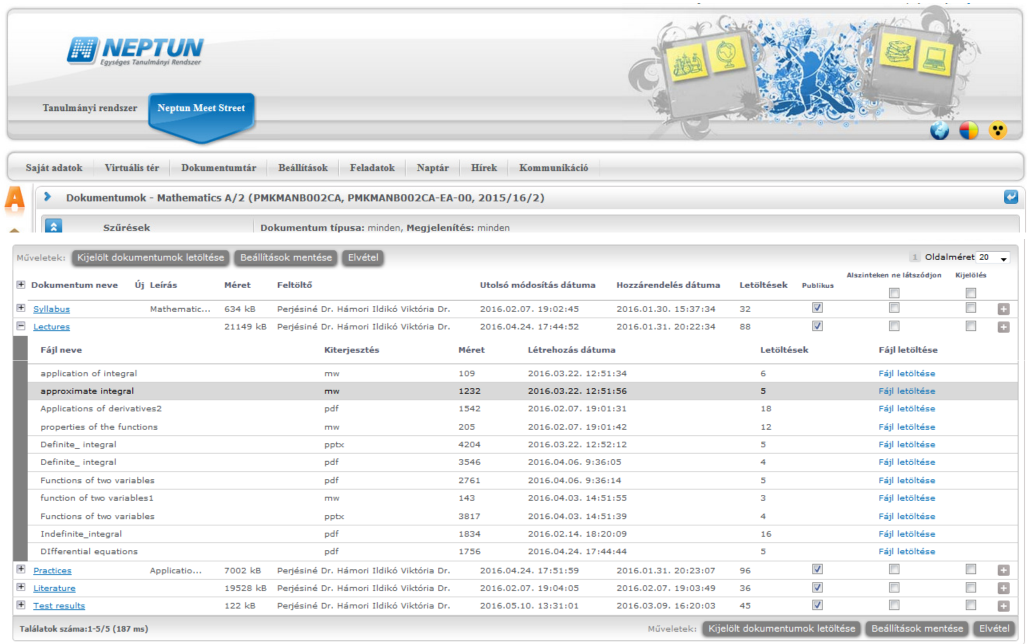
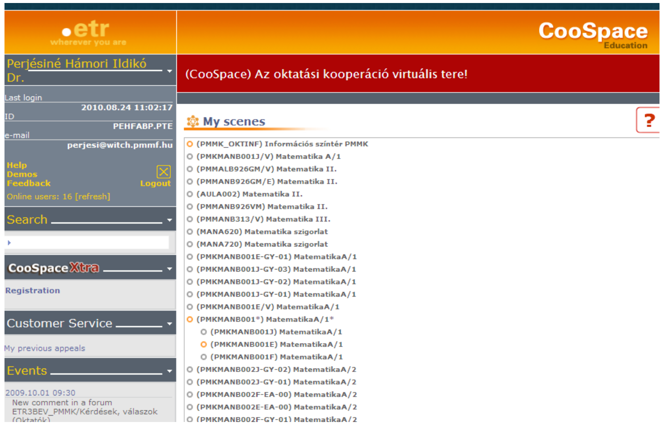
Discovering period







Network based learning

Cover the whole syllabus of the course

Discovering period

From the Newton-law:

```
> eq:=diff(x(t),t$2)+2*beta*diff(x(t),t)+omega^2*x(t)=0;
```

$$eq := \left(\frac{d^2}{dt^2}\,x(t)\right) + 2\,\beta\left(\frac{d}{dt}\,x(t)\right) + \omega^2\,x(t) = 0$$

It's seen that this equation for x(t) is a second-order, constant-coefficient, linear, homogenous differential equation system.

```
> gen_sol:=dsolve(eq,x(t));
```

$$gen\_sol := x(t) = \_C1\,e^{((-\beta+\sqrt{\beta^2-\omega^2})\,t)} + \_C2\,e^{((-\beta-\sqrt{\beta^2-\omega^2})\,t)}$$

Let's check the shape of the given path-time function! Find the particular solution when *x(0)=0, D(x)(0)=vmax*.

```
> part_sol:=dsolve({eq,x(0)=0,D(x)(0)=vmax},x(t));
```

$$part\_sol := x(t) = \frac{1}{2}\,\frac{vmax\,e^{((-\beta+\sqrt{\beta^2-\omega^2})\,t)}}{\sqrt{\beta^2-\omega^2}} - \frac{1}{2}\,\frac{vmax\,e^{((-\beta-\sqrt{\beta^2-\omega^2})\,t)}}{\sqrt{\beta^2-\omega^2}}$$

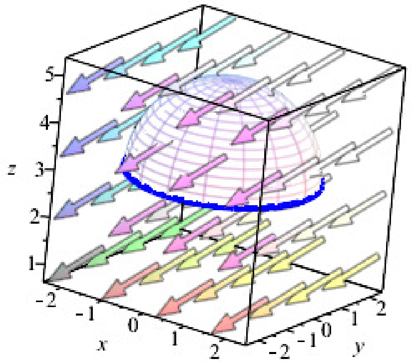
Instrumental orchestration

- Have short textual explanations
- Introduce first every new command - according to the principle of spirality - through a mathematical problem

Discovering period

Surface, curve, field together

```
> abra4:=proc(mezo,felulet,gorbe,u1,u2,v1,v2,t1,t2)
         local rot_mezo,rotabra,feluletabra, gorbeabra;
                   rot_mezo:=curl(mezo,[x, y, z]);
                   rotabra:=fieldplot3d(rot_mezo,x=-2..2,y=-2..2,z=1..5,grid=[4,4,4],arrows=THICK, orientation=
[-48,40]):rotabra;
                   feluletabra:=plot3d(felulet,u=u1..u2,v=v1..v2,axes=boxed,
                   orientation=[6,49], style=hidden, scaling=constrained,numpoints=1000):feluletabra;
                   gorbeabra:=spacecurve(gorbe,t=t1..t2,color=blue,axes=BOX,thickness=4,orientation=[-30,50])
:gorbeabra;
                   plots[display3d]([feluletabra,rotabra,gorbeabra],orientation=[-48,40]);
  end:
> abra4(vektormezo,felgomb,kor,0,2*Pi,0,Pi/2,0,2*Pi);
```



Applications being written according with gradation (white box – black box)

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Discovering period

```
> restart;with(VectorCalculus):
> SetCoordinates( 'cartesian'[x,y,z] );
```

$$cartesian_{x,y,z}$$

```
> field:=VectorField(<x,-z,2*z>);
```

$$field := (x)\bar{e}_x - z\bar{e}_y + 2z\bar{e}_z$$

```
> curve:=<r*cos(t),r*sin(t),t*h/(2*Pi)>;
```

$$curve := (r\cos(t))e_x + (r\sin(t))e_y + \frac{1}{2}\frac{t\,h}{\pi}e_z$$

```
> LineInt(field,Path(curve,t=0..2*Pi),`inert`)=LineInt(field,Path(curve,t=0..2*Pi));
```

$$\int_0^{2\pi}\left(-r^2\cos(t)\sin(t) - \frac{1}{2}\frac{t\,h\,r\cos(t)}{\pi} + \frac{1}{2}\frac{t\,h^2}{\pi^2}\right)dt = h^2$$

```
> Lineintegral:=proc(mezo,gorbe,t1,t2)
      local erinto,skalarszorzat,integrandus,integral,lok;
              erinto:=diff(gorbe,t);
              lok:=subs({x=gorbe[1],y=gorbe[2],z=gorbe[3]},mezo);
              skalarszorzat:=DotProduct(erinto,lok);
              integrandus:=simplify(skalarszorzat);
              integral:=Int(integrandus,t=t1..t2)=int(integrandus,t=t1..t2);
  end:
> Lineintegral(field, curve,0,2*Pi);
```

$$\int_0^{2\pi}\left(-\frac{1}{2}\frac{2r^2\cos(t)\sin(t)\pi^2 + t\,h\,r\cos(t)\pi - t\,h^2}{\pi^2}\right)dt = h^2$$

Instrumental orchestration

- Step by step ⇒ self made procedures ⇒ built in procedures

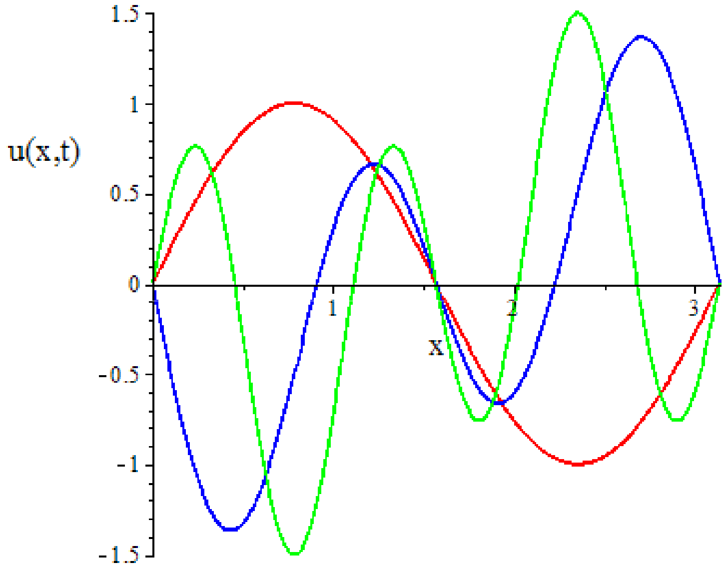
Discovering period

Black box: if we have no time,or students have only few knowledge of the solution of the differential equation, and Fourier series
White box: after the detailed explanation, for experiments, or solving more complicated problem

```
> restart;with(VectorCalculus):with(plots):
> Cord_Eqn := { diff( u(x,t), t, t) = Laplacian( u(x,t), 'cartesian'[x]) };
```

$$Cord\_Eqn := \left\{ \frac{\partial^2}{\partial t^2} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t) \right\}$$

```
> sol:-animate( [[u(x,t) , color=red],
      [-u(x,t)/2-sin(4*x) , color=blue],[-u(x,t)/2+sin(6*x) ,color=green]], t=(
     labels=["x","u(x,t)"], labelfont=[TIMES,ROMAN,14],
     scaling=constrained);
```



Instrumental orchestration

Black box white box: visualization, engineering applications

Discovering period

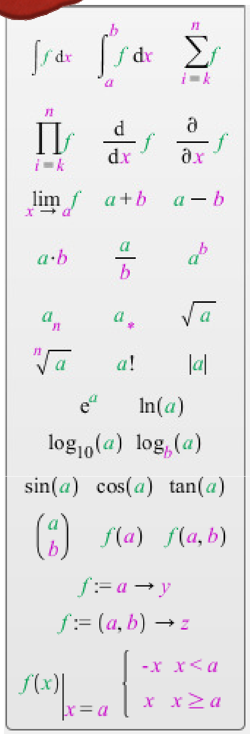
Expectations and observations

- Network based learning +
- Cover the whole syllabus of the course +/-
- Usage as many times as possible +/-
- Instrumental orchestration
  - Have short textual explanations +
  - Introduce first every new command - according to the principle of spirality - through a mathematical problem +
  - Applications being written according with gradation (white box – black box) +
  - Step by step ⇒ self made procedures ⇒ built in procedures +
  - Black box white box: visualization, engineering applications +

Difficulties:

- Deep understanding only for the best students
- Everything is ready: no conceptual understanding
- Didactical problems:
  - Some exercises became routine ones with help of it
  - Not the technical details but the mathematical meaning is always the most important
  - Avoid using CAS only for the end in itself; it is only the inferior of the mathematical subject matter

Period of expanded use



Printed: $\displaystyle\int_2^3 \frac{\sin x}{\cos^2 x} + \sqrt[3]{x}\,\mathrm{d}x$

> $\displaystyle\int_2^3 \frac{\sin(x)}{\cos(x)^2} + \sqrt[3]{x}\,\mathrm{d}x$

```
> int(sin(x)/cos(x)^2+x^(1/3), x = 2 .. 3);
>
```

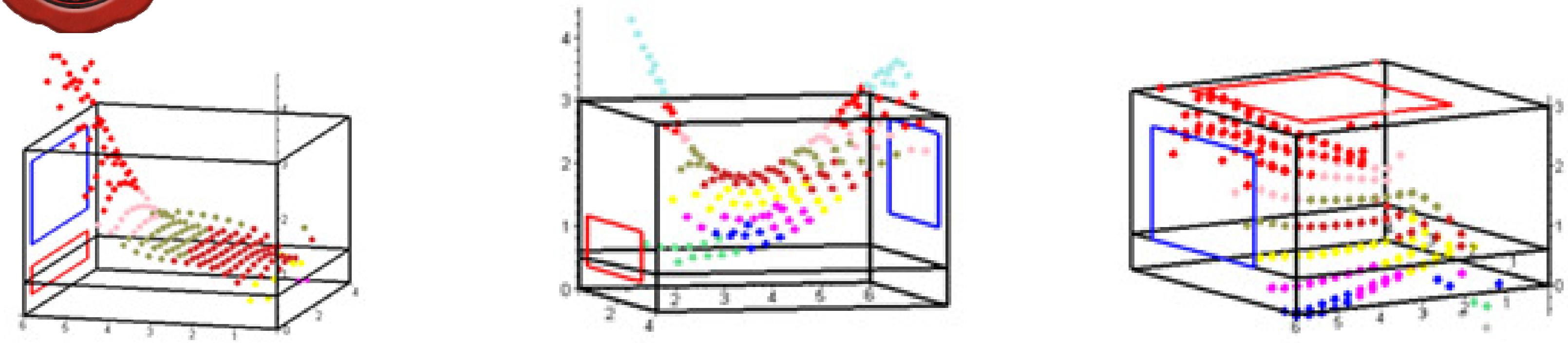
User friendly interface: it is not necessary to prepare everything

Lecture: presentation (ppt, Prezi, video…) definition, theorems, few example + oral explanation

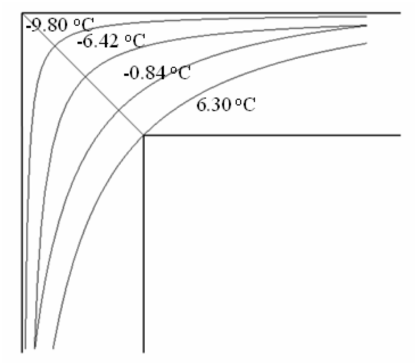
Seminar: paper work, simple examples , more complicated examples using CAS, independent student work

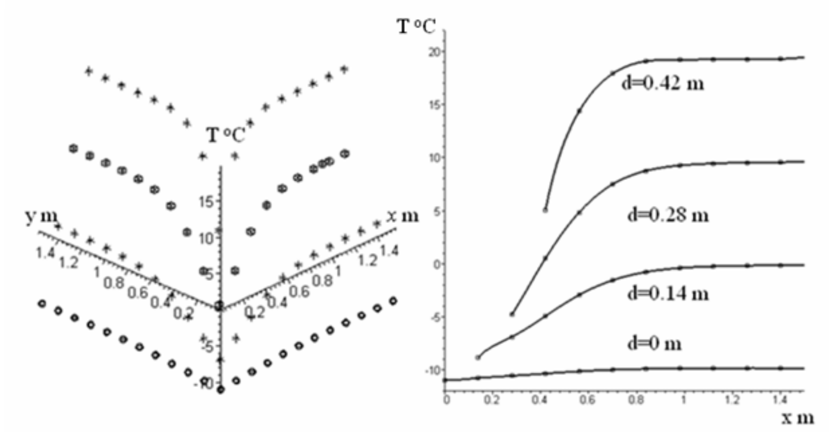
Prepared worksheet for practice class   One expected solution

Period of expanded use



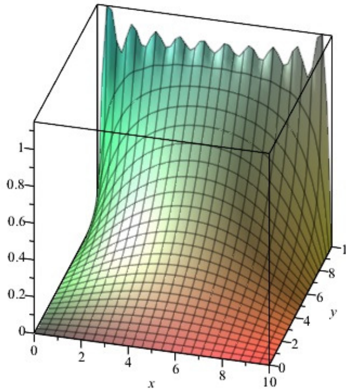
I. Perjési- Hámori: Simulation of Heat Radiation Asymmetry With Maple 7th Vienna Conference on Mathematics Modelling Febr. 15-17, 2012

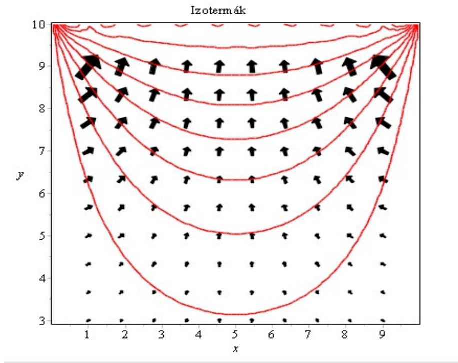




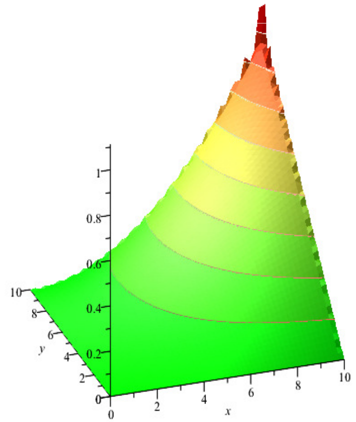
J. Vajda, I. Perjési-Hámori: *Two dimensional mathematical model of heat-transmission of one- and double-layer building* Pollack Periodica Vol. 2, No.3, pp.25-34, 2007.

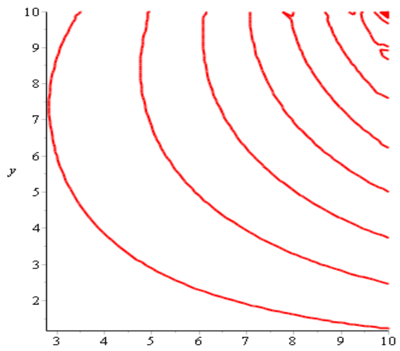
Period of expanded use

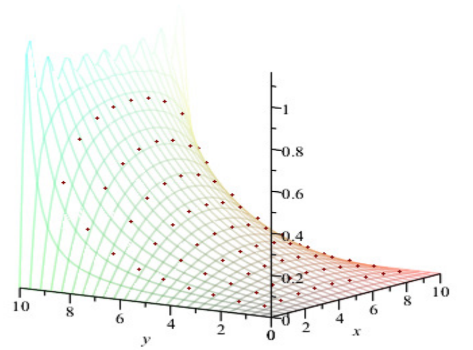





```
> elliptic := proc (n, m, a, b, c, d, f, g)
  local i, j;
  global l, e, u, h, k;
     for j to m − 1 do
     for i to n − 1 do
        l := (j − 1)·(m − 1) + i:
        h := (b − a)/n : k := (d − c)/m :
        if l mod (n − 1) = 1 then u[l − 1] := g(a, c + j·k) fi:
        if l mod (n − 1) = 0 then u[l + 1] := g(b, c + j·k) fi:
        if l -n + 1 ≤ 0 then u[l − n + 1] := g(a + i·h, c) fi:
        if l + n − 1 > (n − 1)·(m − 1) then u[l + n − 1] := g(a + i·h, d) fi:
        e[l] := k^2·(u[l − 1] − 2·u[l] + u[l + 1]) + h^2·(u[l − (n − 1)] − 2·u[l] + u[l
  + (n − 1)]) = f(a + i·h, c + j·k) :
        unassign('u[l − 1]','u[l]','u[l + 1]','u[l − n + 1]','u[l + n − 1]');
     od:
     od:
end proc:
```

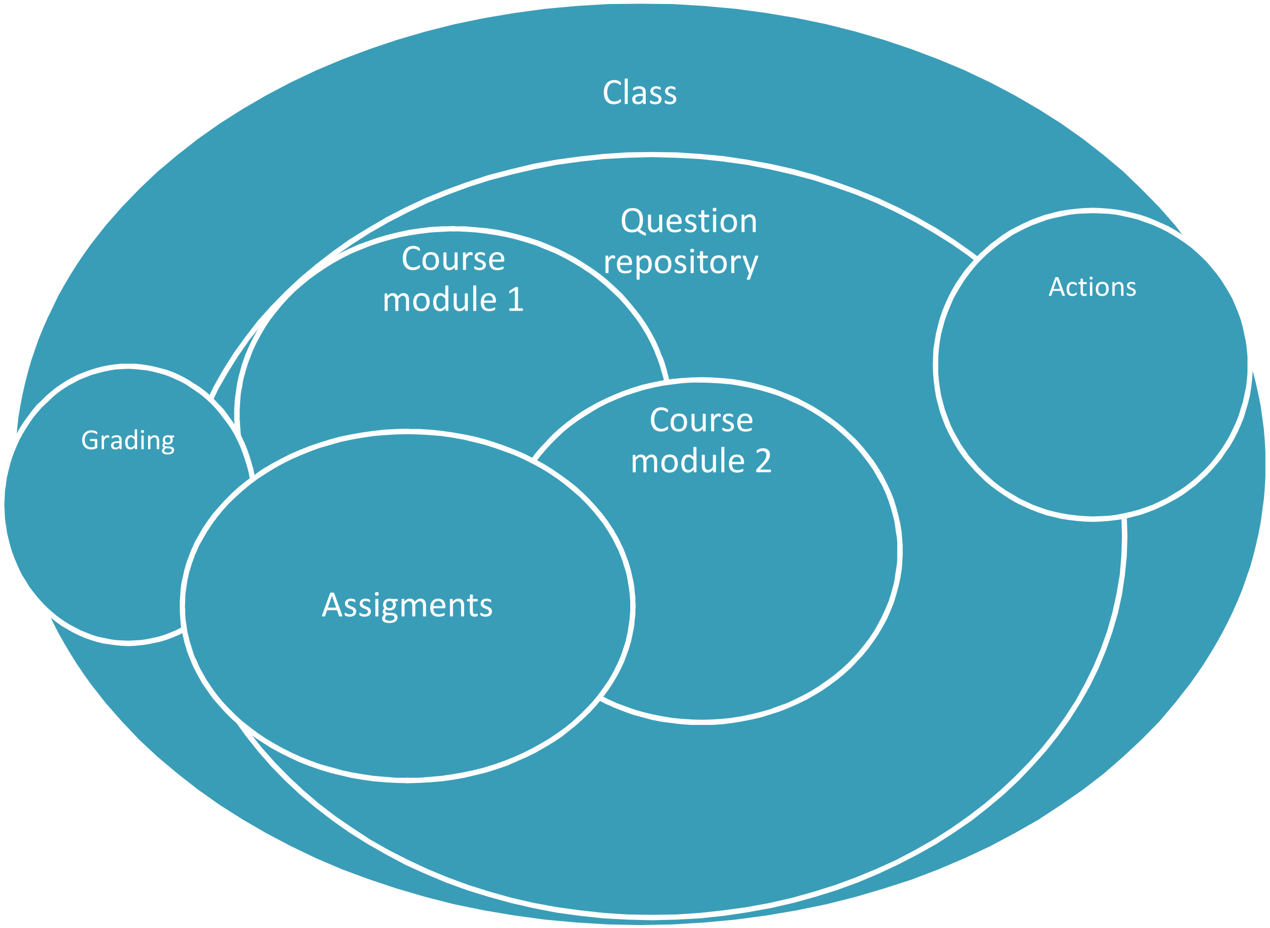






I. Perjési-Hámori: Two Dimensional Mathematical Model of Heat-transmission Using MAPLE poster 8th Vienna Conference on Mathematics Modelling Febr. 17-20, 2015 Mathematical Modelling , Volume # 8 | Part# 1 689-690

Period of expanded use



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Period of expanded use

Actions Content Manager Gradebook System Admin Help

Matematika3

PTE

Ildikó Perjésiné Hámori (perjesi@pmmik.pte.hu)

Select the link for an assignment to begin:

Assignment Name	Points	Type	Availability
Kétváltozós integrál - feladat	24.0	Homework/Quiz	Unlimited
Kétváltozós integál elmélet	8.0	Homework/Quiz	Unlimited
Kétváltozós függvény gradiens és szélsőérték- feladatok	31.0	Homework/Quiz	Unlimited
Kétváltozós függvény parciális és iránymenti derivált-feladatok	32.0	Homework/Quiz	Unlimited
Kétváltozós függvények deriválása-elmélet	5.0	Homework/Quiz	Unlimited
Függvénysor elmélet	10.0	Homework/Quiz	Unlimited
Függvénysorok gyakorló feladatok	12.0	Homework/Quiz	Unlimited
Szamsorok elmelet gyakorlo	10.0	Homework/Quiz	Unlimited
Szamsoros feladatok gyakorlo	30.0	Homework/Quiz	After 4/26/13 9:42 AM

Question 4: (1 points)

A *T* tartományon integrálható $f(x, y)$ kétváltozós függvény $t(T) \neq 0$ területű *T* tartományra vonatkozó integrálközépértékén az $\frac{1}{t(T)} \cdot \iint_T f(x, y)\,dxdy$ kifejezéssel definiált számot értjük.

Határozza meg az $f(x, y) = x + 2y + 2$ függvény *T* tartományra vonatkozó integrálközépértékét, ha a tartományt az x-tengely, az x = 4 egyenes és a $g(x) = 2\sqrt{x}$ függvény grafikonja határolja.

t(T) =

A belső integrál értéke:

A kettős integrál értéke:

Az integrálközépérték:

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Period of expanded use

Question 2: (1 points)

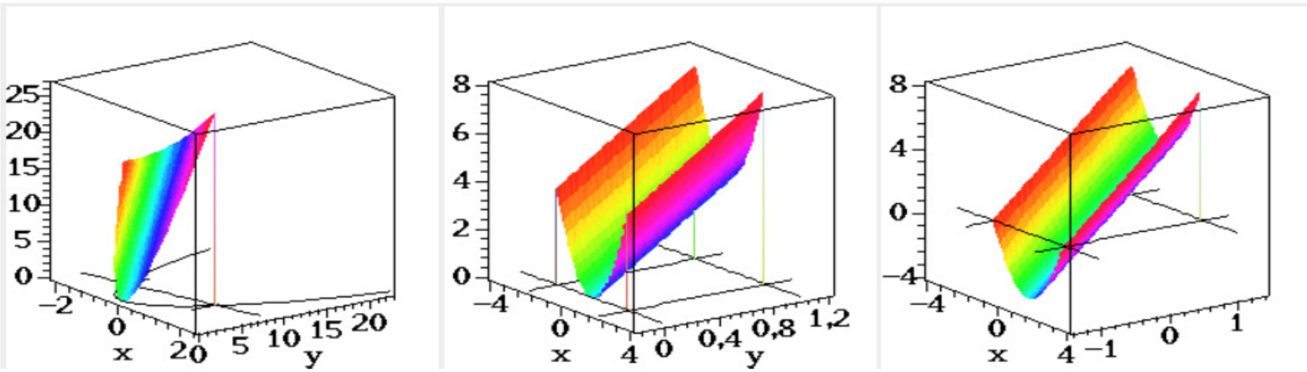
Válassza ki az alábbbi taromány ok közül azokat, amelyek esetén az $\iint_T f(x, y)\,\mathrm{d}T$ integrál az $f(x, y) = x^2 + 4y$ függvény grafikonja és a T tartomány által határolt hengeres térrész térfogatának számértékét adja.

- ☐ $-2 \leq x \leq 2$ és $-1 \leq y \leq 1$
- ☐ $-2 \leq x \leq 2$ és $0 \leq y \leq 1$
- ☐ $y = \sqrt{x}$ és $y = 2x - 1$
- ☐ $y = 4x^2$ és $y = x + 5$ görbék által határolt tartomány.

How did I do?

Comment:

Your response	Correct response	
Válassza ki az alábbbi taromány ok közül azokat, amelyek esetén az $\iint_T f(x, y)\,\mathrm{d}T$ integrál az $f(x, y) = x^2 + 4y$ függvény grafikonja és a T tartomány által határolt hengeres térrész térfogatának számértékét adja. (0%)	Válassza ki az alábbbi taromány ok közül azokat, amelyek esetén az $\iint_T f(x, y)\,\mathrm{d}T$ integrál az $f(x, y) = x^2 + 4y$ függvény grafikonja és a T tartomány által határolt hengeres térrész térfogatának számértékét adja. $-2 \leq x \leq 2$ és $0 \leq y \leq 1$, $y = 4x^2$ és $y = x + 5$ görbék által határolt tartomány.	Incorrect



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Period of expanded use

MathPapa

Problem:

Solve $x + y = 7; x + 2y = 11$

Steps:

I will try to solve your system of equations.

$x + y = 7; x + 2y = 11$

Step: Solve $x + y = 7$ for x:

$x + y + -y = 7 + -y$ (Add -y to both sides)

$x = -y + 7$

Step: Substitute $-y + 7$ for x in $x + 2y = 11$:

$x + 2y = 11$

$-y + 7 + 2y = 11$

$y + 7 = 11$ (Simplify both sides of the equation)

$y + 7 + -7 = 11 + -7$ (Add -7 to both sides)

$y = 4$

Step: Substitute 4 for y in $x = -y + 7$:

$x = -y + 7$

$x = -4 + 7$

$x = 3$ (Simplify both sides of the equation)

Symbolab

$\int \arctan(x)dx$ Go

Graph » Examples »

Solution

« Hide Steps

$\int \arctan(x)dx = x\arctan(x) - \frac{1}{2}\ln|x^2+1| + C$

Steps

$\int \arctan(x)dx$

Show Steps

Apply Integration By Parts: $\int uv' = uv - \int u'v$

$u = \arctan(x),\ u' = \frac{1}{x^2+1},\ v' = 1,\ v = x$

$= \arctan(x)x - \int \frac{1}{x^2+1}x dx$

$= x\arctan(x) - \int \frac{x}{x^2+1}dx$

Show Steps

$\int \frac{x}{x^2+1}dx = \frac{1}{2}\ln|x^2+1|$

$= x\arctan(x) - \frac{1}{2}\ln|x^2+1|$

Add a constant to the solution

$= x\arctan(x) - \frac{1}{2}\ln|x^2+1| + C$

Period of expanded use

integrate sin(x)cos(x)^2

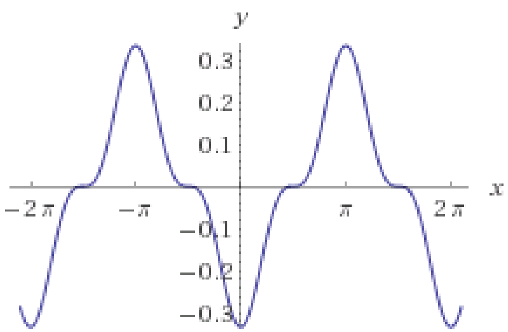
Web Apps | Examples | Random

Indefinite integral:

Step-by-step solution

$$\int \sin(x)\cos^2(x)\,dx = -\frac{1}{3}\cos^3(x) + \text{constant}$$

Plots of the integral:

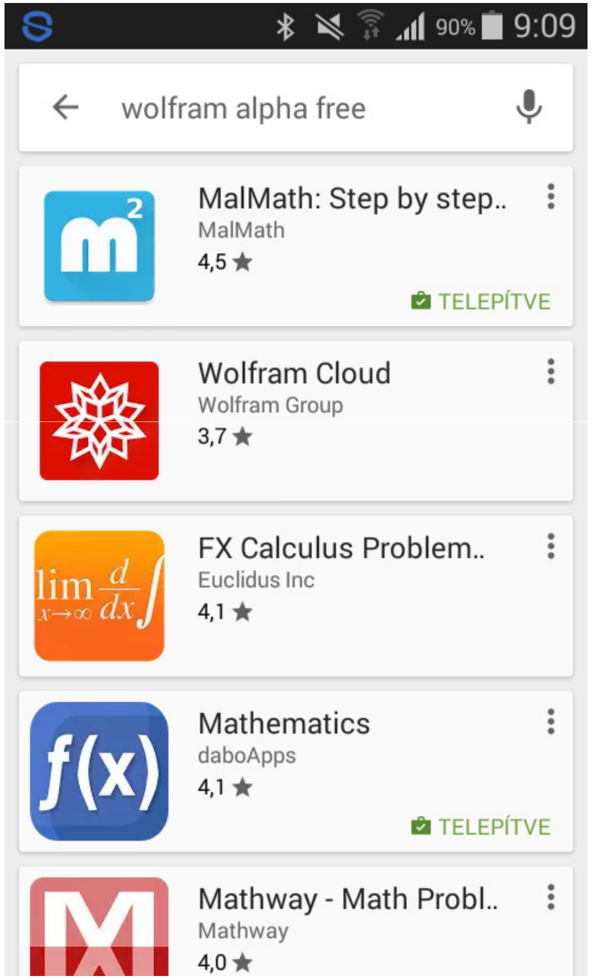


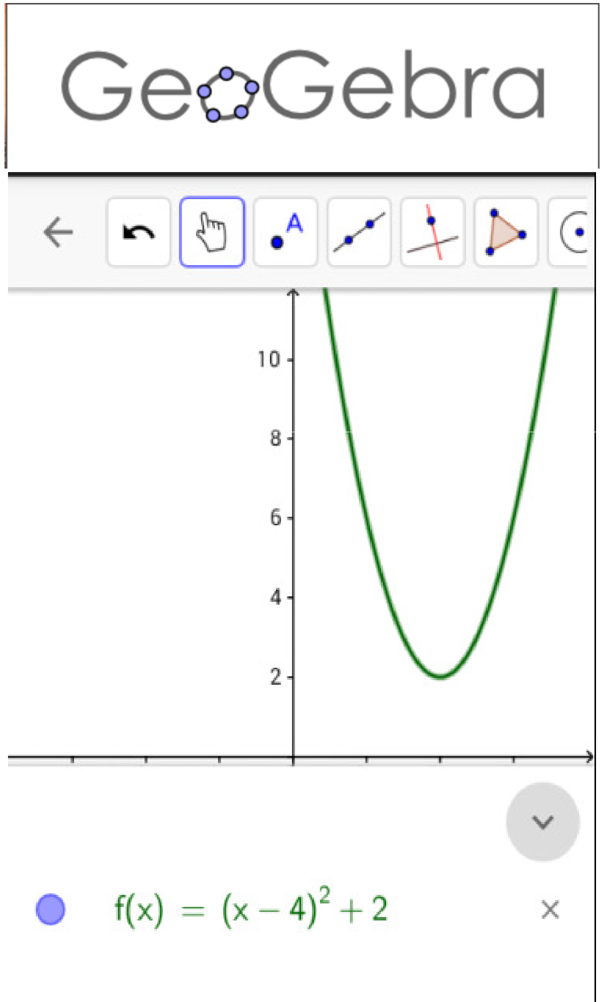
(x from −6.6 to 6.6)

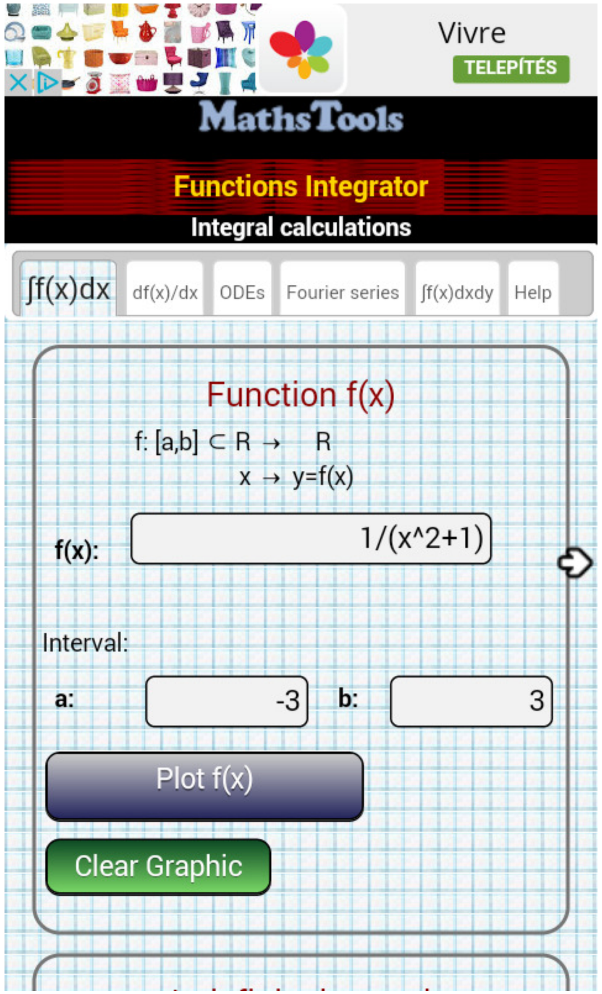
Enable interactivity

Enlarge | Data | Customize | Plaintext | Interactive

Period of expanded use







PÉCSI TUDOMÁNYEGYETEM JUBILEUM 650

Period of expanded use

Expectations and observations:

- Students are aboriginals, teachers are immigrants in IT **+/-**
- Students are users but do not know about programming **-**
- User friendly interface **+**
- Test and assessment system based on Maple **+/-**
- CAS applications for mobile phone, free software's (GeoGebra) **+/-**
- Integrating programing, engineering and math courses (it is the part of every day life) **+**

Difficulties:

- Didactical problems:
    - Role of teacher is not clear
    - Why we have to understand math, why is not enough the applications?
    - There are standards in the softwares, which one is the more useful? (Price, univerity licenses, comparison)

Conclusion

Usage depends on

- Students
- Topics
- Tutor

No comfortable, universal solution.

We have to find the appropriate application forms.

The more we learn more we know about the weakness of our knowledge.

Pécsi Tudományegyetem
JUBILEUM 650

Thank you for your attention